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CRDKNSWC-HD-1427-01 Mathematical Model for a Real Time Ship Maneuvering, Stationkeeping, and Seakeeping Training Simulator

Mathematical Model for a Real Time Ship Maneuvering, Stationkeeping, and Seakeeping Training Simulator

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This report documents the development of the mathematical model of a combined maneuvering, stationkeeping, and seakeeping simulation computer program. The model features a mix of physical based models for the propulsion systems and control surfaces and hydrodynamic coefficient based hull forces. The approach follows the concept of the modular maneuvering model. A quasi-steady assumption is utilized for the seakeeping motion effects. This allows the calm water maneuvering motions to be calculated separately, and the six degree of freedom linear response in waves superimposed. The effects of wind, current, and second order mean drift forces are included.

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Contents

Figures

Notation

A_p	plane area of propeller race over rudder
A_r	planeform area of rudder
A_T	cross sectional area of thruster opening
C_D	airfoil drag coefficient
C_{Dn}	non-dimensional wave drift moment about ship z axis
$C_{D\,rudder}$	rudder drag coefficient, coplanar to rudder centerline
C_{Dx}	non-dimensional wave drift force in direction of ship x axis
C_{Dy}	non-dimensional wave drift force in direction of ship y axis
C_L	airfoil lift coefficient
$C_{L\,rudder}$	rudder lift coefficient, normal to rudder surface
D	propeller diameter
F_n	moment about ship z axis
F_x	force in ship x axis direction
F_y	force in ship y axis direction
g	gravitational acceleration
I_z	yaw mass moment of inertia
J	advance coefficient
J_Q	advance coefficient based on torque
J_T	advance coefficient based on thrust
K_Q	open water propeller torque coefficient
K_T	open water propeller thrust coefficient
L	ship length
L_{cg}	turning radius to ship center of gravity

L_{CL}	turning radius to point on the ship centerline
L_{pt}	turning radius to point off the ship centerline
L_{rud}	turning radius to rudder
m	ship mass
N	revolutions per minute
n	revolutions per second
P/D	propeller pitch/diameter ratio
P_E	effective horsepower (EHP)
Q	torque
R_T	resistance force
R_A	transfer function amplitude
r	yaw rate
$S_\zeta(\omega)$	wave spectra
T	thrust
T_o	wave modal period
T_o'	non-dimensional wave modal period
T_x	ship draft
u	surge velocity, in ship coordinate system
u_c	surge velocity relative to current, in ship coordinate system
u_w	surge velocity relative to wind, in ship coordinate system
V	velocity, general
V_A	speed of advance
V_c	ship velocity magnitude relative to current
$V_{current}$	velocity of current

V_{rud}	velocity at rudder
$\bar{V}_{rud}$	effective velocity over rudder
V_{wind}	velocity of wind
v	sway velocity, in ship coordinate system
v_c	sway velocity relative to current, in ship coordinate system
v_w	sway velocity relative to wind, in ship coordinate system
$x_{p\text{-}r}$	separation distance between propeller and rudder
x_{rud}	rudder distance from ship center of gravity (positive forward)
x_{thrst}	thruster distance from ship center of gravity (positive forward)
X', Y'	earth fixed coordinate system axes
X_{pos}, Y_{pos}	coordinates of ship center of gravity in earth fixed coordinate system
x, y	ship coordinate system axes
x', y'	translating coordinate system axes
y_{pt}	distance of point off the ship centerline (positive to starboard)
y_{sep}	seperation distance between propellers
Φ	blending function for ship speed
Ψ	blending function for relative heading to the waves
β	drift angle
β_c	drift angle relative to current
β_{CL}	drift angle at point on the ship centerline
β_{pt}	drift angle at point off the ship centerline
β_{rud}	drift angle at rudder
Υ	random phase angle
δ	rudder angle

ε	flow straightening angle
ε_p	transfer function phase angle
ζ	wave height
$\zeta_{1/3}$	significant wave height
$\eta_1,\ldots,\eta_6$	surge, sway, heave, yaw, roll, and pitch in seakeeping coordinate system
ρ	water density
$\phi_{current}$	compass angle of direction of current flow
ϕ_{wind}	compass angle of direction of wind
ϕ_{waves}	compass angle of direction of predominate waves
ψ	yaw angle
ω	wave frequency
ω_e	encounter frequency
$1\text{-}w_T$	thrust wake fraction
$1\text{-}w_Q$	torque wake fraction
$1\text{-}t$	thrust deduction factor

Abstract

This report documents the mathematical model of a combined maneuvering, stationkeeping, and seakeeping simulation computer program. The model features a mix of physical based models for the propulsion systems and control surfaces and hydrodynamic coefficient based hull forces. The approach follows the concept of the modular maneuvering model. A quasi-steady assumption is utilized for the seakeeping motion effects. This allows the calm water maneuvering motions to be calculated separately, and the six degree for freedom linear response in waves superimposed. The effects of wind, current, and second order mean drift forces are included.

Administrative Information

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Introduction

The Carderock Division of the Naval Surface Warfare Center (CDNSWC) was tasked by the Coastal Systems Station (CSS), Dahlgren Division, Naval Surface Warfare Center, in Panama City, Florida to develop a simulator model for ship maneuvering and stationkeeping, including seakeeping effects. The simulator is to be incorporated into a team training simulator for the MCM-1 (Avenger) class of mine hunter vessels. Training scenarios will cover all aspects of the MCM mission; mine sweeping, neutralization, and transiting. As such, the operator of the simulated MCM will be required to independently operate both the controllable/reversible pitch propellers, bow thruster, and rudders over the full speed range of the MCM in wind, waves, and current.

The basic physical requirements for the combined maneuvering simulator were to operate in real time on an Intel 80386/80387 personal computer operating at 25 MHz, and be written in the FORTRAN programming language using no machine specific modules. The eventual platform for the simulator program will be a Silicon Graphics model 4440 workstation operating at 40MHz. The source code will be translated by CSS into the ADA programming language for final

inclusion into the team trainer system. In the final form, the simulation will be running at a 16Hz sample rate with approximately 20 to 30 milliseconds allowed for each time step.

In developing a computer program to satisfy the stated requirements, two different starting points were evaluated. First, one of the existing maneuvering programs available could be used and extensive modifications made to it in order to support the zero speed special case for stationkeeping [1,2,3][1]. Second, a new program could be written from scratch. Either option would have to be coupled with a wave induced ship motion calculation methodology. The second option of a new program was selected because intrinsic support for zero speed and reverse and sidewise motions could be built directly into the model. In addition, a new program could be written in a streamlined, minimal form for computational speed, which is easier than trimming down existing program code.

In developing this simulator model, there were several other considerations aside from the basic contractual specifications. The development of this simulator was to be the first opportunity at CDNSWC to develop a maneuvering model based on the modular maneuvering model concept. The modular model concept differs from the traditional approach in several ways. The traditional approach is to use hydrodynamic coefficients derived from a Taylor series expansion of the forces and moments acting on a vessel [4]. The modular modeling approach treats the components of the forces and moments separately. For example, the forces due to the rudder are composed of the basic forces from the airfoil in a flow, the interaction of the hull on the rudder (expressed as flow blockage and straightening effects), and the interaction of the propeller on the rudder (through flow acceleration and straightening due to the propeller race). The forces on the hull would likewise be comprised of the bare hull forces and the interactions of the rudder and propeller on the hull. Implicit in the modular model concept is the use of physical based models for each of the components [5].

The modular maneuvering model offers several advantages over the coefficient based model. Since each component effecting maneuvering performance is separate in the modular model, changes can be isolated. This includes changes to program source code and model as well

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[1]References in brackets are listed at the end of this report.

as changes to the ship design. For example, the effect of a different size or shape rudder is isolated to the rudder model and the interactions on other components. Hence, the hull force model or propeller force model remain unchanged. This is particularly useful in preliminary design where different concepts are investigated. In later phases of design, model test data may be available for a baseline design. The design can then be modified with different propellers or rudders without necessarily invalidating the previous model test work.

By using the modular maneuvering model concept for this simulator, the program could be written in a highly structured manner with clear divisions between force and moment contributors. This facilitates better internal documentation of the source code and allows easier maintenance and future expansion of capabilities.

This report documents the development of the combined maneuvering, stationkeeping, and seakeeping simulation mathematical models used in the computer program. A separate programmers manual documents the computer program source code as implemented for the MCM class[a].

Combined Maneuvering, Stationkeeping, and Seakeeping

In developing a time domain model of ship maneuvering in waves, there are several problems to be considered. There are frequency dependent coefficients in the equations of motion, unsteady ship speed and heading, effects of the past history of a body's motion (memory effects), and first and second order wave induced oscillatory exciting forces. Previous attempts to model ship maneuvering in waves have generally focused on the specific problems of course keeping ability, or capsizing and broaching. Few complete models that can represent general maneuvering in waves have been developed, with even fewer implementations of those models [6]. In general, such models are large, cumbersome, expensive, and not well suited to preliminary design applications or real time, man-in-the-loop simulations.

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[a] In a report of higher classification.

The approach taken here is to decouple the relatively slow maneuvering motions from the relatively faster seakeeping motions. This is accomplished by calculating the calm water maneuvering motions and adding the linear seakeeping responses. The advantage of this method is that the seakeeping motions can be pre-computed outside of the maneuvering simulation. Figure 1 shows the conceptual model that was developed as a framework for the development of the components of the model.

It is important to note that estimation or experimental determination of the hydrodynamic characteristics of each component of the ship (hull, rudders, props, etc.), it is assumed that the body is moving with sufficient speed and has sufficient surface roughness to insure that the flow is fully turbulent. However, the simulation model has the capability to operate at very low speeds where the flow could be laminar. No attempt is made in this model to account for hydrodynamic differences between laminar and turbulent conditions.

Coordinate Systems

Within this simulator model, a multitude of coordinate systems and frames of reference are employed. The standard coordinate systems for ship motions and maneuvering are different from each other by a 180 degree rotation about the ship's x axis. The bottom of Figure 2 shows the orientation of both coordinate systems [4]. In this work, the standard maneuvering coordinate system will be used unless explicitly stated. Figure 2 shows the translating ship coordinate system (x',y'), the earth fixed coordinate system (X',Y'), and the ship reference coordinate system (x,y). The positive sense of the rudder and drift angles and propeller and bow thruster thrust are also shown. The origin of the ship coordinate systems are at the ship's center of gravity. Current and wind speed and direction and predominate wave direction are specified in earth fixed coordinate system using compass heading angles.

Maneuvering Equations of Motion

The general, nonlinear equations of motion for a maneuvering ship in the horizontal plane (without roll) are given in Reference 4.

$$
\begin{aligned}
m(\dot{u} - v\psi) &= \Sigma F_x \\
m(\dot{v} + u\psi) &= \Sigma F_y \\
I_{zz}\psi &= \Sigma M_z
\end{aligned}
\tag{1}
$$

The cross product terms on the left side of the x and y equations are the result of these equations being written in the ship coordinate system. Rewriting these as a set of first order differential equations,

$$
\begin{aligned}
\dot{u} &= \tfrac{1}{m}(vr + \Sigma F_x) \\
\dot{v} &= \tfrac{1}{m}(-ur + \Sigma F_y) \\
\dot{r} &= \tfrac{1}{I_z}\Sigma F_n
\end{aligned}
\tag{2}
$$

where ψ has been replaced by r.

Relating the ship coordinate system back to the earth fixed coordinate system yields another set of first order differential equations,

$$
\begin{aligned}
\dot{X}_{pos} &= u \sin\psi + v \cos\psi \\
\dot{Y}_{pos} &= u \cos\psi - v \sin\psi
\end{aligned}
\tag{3}
$$

Equations 2 and 3, when taken together, form a set of five, first order nonlinear state equations. The general form is given by

$$\dot{X} = A(X) + B(U,X) \tag{4}$$

Solution of the time evolution of these equations is performed by first order Euler time step,

$$X^{n+1} = X^n + [A(X^n) + B(U^n, X^n)]\Delta t \tag{5}$$

The first order Euler method is generally stable (for dynamically stable ships) and will not propagate errors if the time step size is small compared to the speed of ship maneuvering motions.

External Forces

If we disregard the seakeeping aspects of the problem for now, the total external forces and moments acting on a maneuvering ship are represented by ΣF_x, ΣF_y, and ΣF_n. These summations can be expanded into components for the hull, rudders, propellers, thrusters, wind, and waves [4].

$$\Sigma F_x = F_{x_{hull}} + F_{x_{rudder}} + F_{x_{props}} + F_{x_{thruster}} + F_{x_{wind}} + F_{x_{wave}}$$
$$\Sigma F_y = F_{y_{hull}} + F_{y_{rudder}} + F_{y_{props}} + F_{y_{thruster}} + F_{y_{wind}} + F_{y_{wave}}$$
$$\Sigma F_n = F_{n_{hull}} + F_{n_{rudder}} + F_{n_{props}} + F_{n_{thruster}} + F_{n_{wind}} + F_{n_{wave}} \quad (6).$$

Current Effects

Hull forces are due to the rotation and translation of the vessel through the water. In order to include the effects of a current, the relative velocity between the ship and the current must be calculated.

$$u_c = V\cos\beta + V_{current}\cos(\phi_{current} - \psi)$$
$$v_c = -V\sin\beta - V_{current}\sin(\phi_{current} - \psi) \quad (7)$$

where the ship velocity and drift angle are

$$V = \sqrt{\dot{X}_{pos}^2 + \dot{Y}_{pos}^2}$$
$$\beta = \psi - \arctan\frac{\dot{X}_{pos}}{\dot{Y}_{pos}} \quad (8)$$

and the drift angle is considered zero if both the X_{pos} and Y_{pos} velocities are zero. The velocity and drift of the ship relative to the current is therefore

$$V_c = \sqrt{u_c^2 + v_c^2}$$
$$\beta_c = \arctan\frac{-v_c}{u_c} \quad (9).$$

Using the relative velocity through the water requires that the surge and sway equations in equation 2 be rewritten as

$$\dot{u}_c = \frac{1}{m}(v_c r + \Sigma F_x)$$
$$\dot{v}_c = \frac{1}{m}(-u_c r + \Sigma F_y) \quad (10).$$

Equation 3 must then be corrected for the current and written as

$$\dot{X}_{pos} = \hat{u}\sin\psi + \hat{v}\cos\psi$$
$$\dot{Y}_{pos} = \hat{u}\cos\psi - \hat{v}\sin\psi \quad (11)$$

where

$$\hat{u} = u_c - V_{current}\cos(\psi - \phi_{current})$$
$$\hat{v} = v_c + V_{current}\sin(\psi - \phi_{current}) \quad (12).$$

Hull Force Model

As stated in the introduction, the traditional approach to modeling hull forces has been to assume the linearity of forces and expand them with a Taylor series expansion. This is still the most straight forward approach to developing hull forces as a function of surge, sway, and yaw motions. Here we will not include the effects of rudder, props, or bow thrusters in the expansion by considering the bare hull only.

If the hull forces are considered to be functions of surge, sway, and yaw motion only , we may write

$$
\begin{aligned}
F_{x_{hull}} &= \frac{\partial F_x}{\partial v}v+\frac{\partial F_x}{\partial v^2}v^2 + \frac{\partial F_x}{\partial r}r+\frac{\partial F_x}{\partial r^2}r^2 + \frac{\partial F_x}{\partial \dot{v}}\dot{v}+\frac{\partial F_x}{\partial \dot{r}}\dot{r} \\
F_{y_{hull}} &= \frac{\partial F_y}{\partial v}v+\frac{\partial F_y}{\partial v^2}v^2 + \frac{\partial F_y}{\partial r}r+\frac{\partial F_y}{\partial r^2}r^2 + \frac{\partial F_y}{\partial \dot{v}}\dot{v}+\frac{\partial F_y}{\partial \dot{r}}\dot{r} \\
F_{n_{hull}} &= \frac{\partial F_n}{\partial v}v+\frac{\partial F_n}{\partial v^2}v^2 + \frac{\partial F_n}{\partial r}r+\frac{\partial F_n}{\partial r^2}r^2 + \frac{\partial F_n}{\partial \dot{v}}\dot{v}+\frac{\partial F_n}{\partial \dot{r}}\dot{r}
\end{aligned}
\qquad (13)
$$

where the higher order terms have been omitted. The derivatives, also known as hydrodynamic or maneuvering coefficients, are usually written in a shorthand notation such that $\frac{\partial F_x}{\partial v} = X_v$, $\frac{\partial F_y}{\partial v} = Y_v$, and $\frac{\partial F_n}{\partial v} = N_v$, for example. The acceleration terms $Y_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{v}}$, and $N_{\dot{r}}$ basically represent low frequency added mass effects [4].

Hydrodynamic coefficients can be obtained from captive model tests, estimation, or system identification from full scale trials or free running model experiments. Each of these methods has their own merits and drawbacks. A discussion of these is beyond the scope of this document and the reader is referred to the references [4,7,8,9].

The normal practice is to express the hydrodynamic coefficients in non-dimensional form, for example

$$
Y'_v = \frac{Y_v}{\frac{1}{2}\rho L^2 V_s^2} \qquad (14).
$$

Here, the velocity in the denominator is the relative velocity through the water. When a non-dimensional hydrodynamic coefficient is used to calculate the non-dimensional force or moment, it implies that the associated variable (v in the example case of equation 14) to be multiplied is also in its non-dimensional form. Hence, the dimensional force is obtained from

summing the products of the non-dimensional coefficients and associated non-dimensional variables, and then multiplying by the factor $\frac{1}{2}\rho L^2 V_c^2$ (L^3 for moments)

Within the series expansion for the x force, a term has been added to account for the hull resistance. The effective power (EHP) curve is represented as a polynomial curve from which the hull resistance is obtained,

$$P_E(V) = V^3\left[c_1 V + c_2 V^2 + c_3 V^3\right]$$
$$R_T = \frac{550 P_E}{V} \qquad (15).$$

The polynomial coefficients are obtained from curve fits of upright straight line (zero drift angle) resistance tests [10]. The ship speed used in this implementation will be x component of the relative speed through the water (u_c). In the special case of zero ship speed, the resistance is zero.

Rudder Force Model

The rudder is the primary control surface by which directional control is maintained. In order to determine the forces acting on the rudder, the actual flow velocity and direction at the rudder must be determined from the general motion of the ship. Figure 3 shows a schematic representation of a ship in a turn. If the velocity components of the ship center of gravity (cg) and the yaw rate are known then the drift angle, turning radius, and inflow angle at the rudder is calculated from

$$L_{cg} = V_c / r$$
$$L_{rud} = \mathrm{sign}(r)\sqrt{(x_{rud}\cos\beta_c)^2 + (L_{cg} - x_{rud}\sin\beta_c)^2}$$
$$\beta_{rud} = \beta - \mathrm{sign}(r)\arctan\left(\frac{x_{rud}\cos\beta_c}{L_{cg} - x_{rud}\sin\beta_c}\right)$$
$$V_{rud} = |rL_{rud}|$$
$$\alpha = \beta_{rud} + \delta - \varepsilon \qquad (16).$$

In the case of $r=0$ and $V \neq 0$, then the turning radius is infinite and $\beta_{rud} = \beta_c$ and $V_{rud} = V_c$. The rudder angle is given by the commanded rudder angle up to the point that a set maximum rudder rate is achieved.

When the point is not on the ship centerline, an additional correction is required. If the turning radius and drift angle of a point on the centerline is L_{CL} and β_{CL}, then the turning radius and drift angle of a point off the centerline is

$$L_{pt} = \mathrm{sign}(L_{CL})\sqrt{\left(y_{pt}\sin\beta_{CL}\right)^2 + \left(L_{CL} - y_{pt}\cos\beta_{CL}\right)^2}$$
$$\beta_{pt} = \beta_{CL} + \mathrm{sign}(L_{CL})\arctan\left(\frac{y_{pt}\sin\beta_{CL}}{L_{CL}-y_{pt}\cos\beta_{CL}}\right) \quad (17).$$

The correction, ε in equation 16, is a flow straightening coefficient which represents the blockage of the hull to the flow. It has been seen that the flow straightening is primarily dependent on the geometric drift angle at the rudder, β_{rud} [11]. A relation derived from the data presented in Reference 11 is employed,

$$\varepsilon = c_0 + c_1\beta_{rud} + c_2\beta_{rud}^3 + c_3\beta_{rud}^5 \quad (18)$$

where it is assumed that $\varepsilon = 0$ if $|\beta_{rud}| \geq 90$ degrees.

There is an additional interaction effect on the effective flow velocity when the propeller race is directed over the rudder [12],

$$\bar{V}_{rud}^2 = \frac{A_p}{A_r}[(1-w_T)V_{rud}\cos\beta_{rud} + kV_\infty]^2 + \frac{A_r-A_p}{A_r}(1-w_T)^2V_{rud}^2\cos^2\beta_{rud} \quad (19)$$

where

$$V_\infty = -(1-w_T)V_{rud}\cos\beta_{rud} + \sqrt{(1-w_T)^2V_{rud}\cos^2\beta_{rud} + \tfrac{8}{\pi}K_Tn^2D^2}$$
$$k = 0.5 + 0.26527\tanh(1.2775\xi) + 0.17533\tanh(2.555\xi)$$
$$\xi = 2x_{p\text{-}r}/D \quad (20).$$

Some portion of the propeller race will be directed over the rudder when: $u_c>0$ and $p/d>0$, and possibly $u_c<0$ and $p/d>0$.

The forces acting on the rudder are given by the lift and drag coefficients. As the rudder will operate in a diversity of flow conditions, lift and drag characteristics are needed for forward, reverse, and sideways flow [13]. Like the hull hydrodynamic coefficients, rudder lift and drag coefficients are generally determined in a fully turbulent flow condition. These coefficients will be used at all times in the simulator model even if the flow speed and geometry at either port or starboard rudder would in fact be indicative of laminar flow.

Lift and drag are defined respectively as parallel and normal to the flow. In order to obtain the forces acting on the hull, a transformation is performed from the flow reference frame to the rudder reference frame and finally to the moving ship's coordinate system. This is done using

$$C_{L_{rudder}} = C_L \cos\alpha + C_D \sin\alpha$$
$$C_{D_{rudder}} = C_D \cos\alpha - C_L \sin\alpha \tag{21}$$

and

$$F_x = (C_{L_{rudder}} \cos\delta - C_{D_{rudder}} \sin\delta)\left(\tfrac{1}{2}\rho \bar{V}_{rud}^2 A_r\right)$$
$$F_y = (-C_{D_{rudder}} \cos\delta - C_{L_{rudder}} \sin\delta)\left(\tfrac{1}{2}\rho \bar{V}_{rud}^2 A_r\right)$$
$$F_n = F_y * x_{rud} \tag{22}$$

Though the rudders on the MCM operate together, each will experience a different inflow geometry and potentially different propeller races. Therefore, the forces and moments should be calculated for each rudder separately and summed together. The machinery model for the steering gear imposes limits on the extremes of position and the maximum rate of change of position.

Propeller Force Model

The propellers are the primary means for the ship operator to control the speed of his vessel. They also provide some directional control when the two propellers are operated individually at different speeds and/or pitch settings.

Since the MCM has controllable reversible pitch (CRP) propellers, the full four quadrant open water propeller performance at all pitch settings must be modeled. The open water propeller data is represented as curves of thrust and torque coefficients (K_T and K_Q respectively) over a range of negative and positive advance coefficient, J [14]. These are defined as

$$J = \frac{V_a}{nD}\text{, and } K_T = \frac{T}{\frac{1}{2}\rho n^2 D^4}\text{, and } K_Q = \frac{Q}{\frac{1}{2}\rho n^2 D^5} \tag{23}$$

where V_a is the speed of advance through the water, n is the propeller speed, D is the propeller diameter, T is the propeller thrust, and Q is the propeller torque. For the purposes of this model, the speed of advance is

$$V_a = V(1 - w_T) \tag{24}$$

where $(1-w_T)$ is the thrust wake fraction and V is the x component of the velocity of a point at the propeller hub (refer to the discussion in the previous rudder force section). The thrust wake fraction represents a blockage effect of the hull on the flow to the propeller and is usually

determined by powering experiments. In straight line motion (zero drift angle), the wake fraction is the measured value from experiments [10]. However, as the ship maneuvers and operates at non-zero drift angles, there will be less blockage by the hull on the flow to the propellers. For inflow angles less than 90 degrees, a cosine squared correction is used,

$$(1-w_T) = \sin^2\beta_{prop} + (1-w_T)_{(\beta_{prop}=0)}\cos^2\beta_{prop} \tag{25}$$

and $(1\text{-}w_T)=1$ for angles greater then 90 degrees. Here β_{prop} is the drift angle of a point at the propeller hub. Since the propellers are operated independently from each other, the V in equation 24 and β_{prop} in equation 25 should be calculated for both port and starboard propellers.

A polynomial curve fit for each K_T and K_Q curve is made for each pitch/diameter (P/D) value and both positive and negative J. Reference [15] gives twelfth order polynomial coefficients for the MCM design propeller. In order to obtain the operating values for either port or starboard propeller, the polynomials are evaluated at the appropriate value of advance coefficient. Linear interpolation is used between curves to the actual propeller pitch/diameter setting. The engine model (to be discussed later) determines the rpm to be used in calculating J for the port and starboard props.

Having the total thrust from the K_T values, it must be decreased by using the thrust deduction factor. Thrust deduction is corrected for non-zero inflow angles in a similar manner to that used on the wake fraction in equation 25. Propeller thrust acts only along the longitudinal direction of the ship hull. No oblique flow effect are included except for the wake fraction and thrust deduction factor corrections. As a result, and by assumption, no side forces are exerted. The forces and moments delivered by the propellers to the ship are given by,

$$\begin{gathered} F_x = K_T(1\text{-}t)\left(\tfrac{1}{2}\rho n^2 D^4\right) \\ F_y = 0.0 \\ F_n = \pm\frac{y_{sep}}{2}F_x \end{gathered} \tag{26}$$

For the yaw moment, the positive sign is taken for the port propeller and the negative for the starboard propeller.

Bow Thruster Force Model

The bow thruster is an auxiliary device for providing directional control. The MCM class of vessels use the Omni Thruster™ bow thruster. This device is different from the more conventional tunnel type thrusters. The Omni Thruster™ uses a sea chest in the keel for a water inlet and then splits the flow to either a port or starboard side outlet. For the purposes of this simulator, a relatively simple model will be used.

The ship operator will issue a thruster command of between +100% and -100% thrust. A second order polynomial is used to transform the command setting to the full scale thrust. The data needed to perform the curve fit is usually provided by the thruster manufacture and should assume no forward speed. Forward speed degradation effects have been studied by Chislett and Bjorheden [16] and McCreight has performed curve fits of the published data [12]. The thruster forces can be determined from

$$\begin{aligned} F_x &= 0.0 \\ F_y &= T[\exp(-13.3m^2) + 0.627396m - 0.385772m^2 + 0.124873m^3] \\ F_n &= [T + (1 - \tfrac{2m}{3})(F_y - T)]x_{thruster} \end{aligned} \tag{27}$$

where

$$m = \frac{V_e \cos\beta_e}{\sqrt{T/\rho A_T}} \tag{28}$$

and T is the thruster thrust with no ship velocity. It is assumed that m=0 if T=0. For a ship moving aft, it is assumed that there is negligible degradation. This is represented here by m=0 for $|\beta_c| > 90$ degrees. The velocity and drift angle at the thruster should also be calculated for the ship in a turn using the method outlined in equation 16.

Wind Force Model

In a manner similar to the current and relative speed through the water, a transform between the wind speed and direction in the earth fixed reference system and the ship reference system is given by

$$
\begin{aligned}
u_w &= V\cos\beta + V_{wind}\cos(\phi_{wind}-\psi) \\
v_w &= -V\sin\beta - V_{wind}\sin(\phi_{wind}-\psi) \\
V_w &= \sqrt{u_w^2+v_w^2} \\
\beta_w &= \arctan\frac{v_w}{u_w}
\end{aligned}
\qquad (29).
$$

The sway force and moment coefficients are defined as the side force and yaw moment (non-dimensionalized) acting on the ship when the wind is blowing across the port beam. The surge force and moment coefficients are defined when the wind is blowing across the bow. The total force and moment contribution from the wind is a blended value of the two sets of wind force and moment coefficients [12],

$$
\begin{aligned}
F_x &= \tfrac{1}{2}\rho_{air}V_w^2(Area_x C_{x_{wind}}\cos\beta_w) \\
F_y &= \tfrac{1}{2}\rho_{air}V_w^2(Area_y C_{y_{wind}}\sin\beta_w) \\
F_n &= \tfrac{1}{2}\rho_{air}V_w^2(Area_x C_{xz_{wind}}\cos 2\beta_w + Area_y C_{yz_{wind}}\sin 2\beta_w)
\end{aligned}
\qquad (30).
$$

The negative signs are due to the wind drag opposing forward motion.

Wave Drift Force Model

The wave drift forces being addressed here are the mean value, second order wave drift forces. This is an acceptable model for use in real time man-in-the-loop simulation where calculation of the full time varying second order forces would be burdensome. The mean wave drift forces are represented by

$$
\begin{aligned}
F_x &= \rho g B \zeta_{1/3}^2 C_{D_x}(T'_o, \phi_{wave}) \\
F_y &= \rho g L \zeta_{1/3}^2 C_{D_y}(T'_o, \phi_{wave}) \\
F_n &= \rho g L^2 \zeta_{1/3}^2 C_{D_z}(T'_o, \phi_{wave})
\end{aligned}
\qquad (31)
$$

where

$$
T'_o = T_o\sqrt{g/T_x} \qquad (32)
$$

is the non-dimensional wave modal period [12].

McCreight and Jones have investigated the wave drift forces on an MCM model in uni-directional regular seas [17]. Since then, McCreight has developed an unpublished interpolation method for the original model test data. The non-dimensional wave drift force

coefficients used in the simulation computer program are determined by interpolation of the test data.

Engine Machinery Model

The USS AVENGER (MCM-1) is powered by four Waukesha LN 1616 DSIN diesel engines geared to two Transamerica-Delaval reduction gears. There are also twin light load electric motors that can be used. This simulator model will assume that two diesel engines driving one shaft are represented by one single large engine driving its own shaft with no cross coupling between shafts. No model for the light load motors is included.

A basic model for a diesel engine is a constant torque model [18]. Within the implementation of the constant torque model, engine throttle settings are given as 0 to 100% such that 100% throttle produces maximum engine torque. Limits are placed on the rate of change in produced engine torque. In addition, the shaft speed is limited to a maximum value as is the rotational acceleration of the shaft. The rate of change of propeller pitch is also limited to a set maximum value [19].

For a constant torque, the shaft speed is not specified and must be backsolved using known characteristics of the propeller and the flow conditions at the propeller. Since this model must perform stationkeeping as well as general maneuvering, there are several special cases that must be accounted for. In addition, since reversible propellers are used, the shaft speed should be prevented from becoming negative. The trivial case of zero torque (corresponding to zero throttle setting) produces zero shaft speed.

The bollard pull condition exists when there is a non-zero torque and zero ship speed. Here, the advance coefficient J is also zero. The values of torque and thrust coefficients (K_Q and K_T) are found directly from the open water propeller characteristics at a given propeller pitch setting, interpolating between discrete pitch values as necessary. The shaft speed is then given by,

$$n = \sqrt{Q/\left(\rho K_Q D^5\right)} \qquad (33).$$

The final condition exists when both the torque and ship speed are non-zero. In this case the factor,

$$\frac{K_Q}{J_Q^2} = Q/\left(\rho D^3 V_s^2\right) \tag{34}$$

is computed. This is independent of shaft speed. Knowing the open water propeller characteristics as a polynomial function of advance coefficients, we can write with the use of equation 34,

$$0 = c_0 + c_1 J_Q + c_2 J_Q^2\left(1 - Q/\left(\rho D^3 V_s^2\right)\right) + c_3 J_Q^3 + \ldots + c_{12} J_Q^{12} \tag{35}$$

The real root (or roots) is the torque advance coefficient at the current condition of ship speed, shaft torque, and propeller pitch. The thrust advance coefficient is given from,

$$J_T = J_Q \frac{1-w_T}{1-w_Q} \tag{36}$$

The propeller open water thrust coefficient can be obtained from the known J_T and the polynomial curve for K_T. The shaft speed is given by,

$$n = \frac{V_s}{D\, J_T} \tag{37}$$

or equation 33 if $J_T = 0$. If the shaft speed or rate of change of shaft speed is found to exceed preset limits, then it is appropriately down graded to fall within the limits. Since J_T and J_Q were found from relations independent of shaft speed, those values will not change, hence K_Q will not change either. However, the engine torque will change, and its new value is computed from,

$$Q = K_Q\left(\rho n^2 D^5\right) \tag{38}$$

Seakeeping Effects

The six degree of freedom responses are governed by frequency domain transfer functions, calculated using a linear ship motions computer program [20]. Time histories for the six degrees of freedom are generated from the amplitude and phase information contained within the transfer functions,

$$\eta_i(t) = \sum_k \left[(R_{A_k} \cdot \zeta_k)\cos(\omega_{e_k} t + \gamma_k + \varepsilon_{r_k})\right] \tag{39}$$

This gives the motion η at time t where the summation is over discrete frequency values, R_A and ε_P is the transfer function amplitude and phase at the k^{th} wave frequency, ω_e is the frequency of encounter at the k^{th} wave frequency, and ζ_k is the wave height at the k^{th} wave frequency defined by

$$\zeta_k = \sqrt{2\int_{\omega_k-d\omega/2}^{\omega_k+d\omega/2} S_\zeta(\omega)d\omega} \qquad (40).$$

A uniformly distributed random phase angle, γ_k, is included. Equation 39 applies specifically to long crested seas. Short crested sea responses require an additional summation over wave direction [21].

There are 576 individual six degree of freedom time histories used. They are comprised of six ship speeds (0 to 10 knots in 2 knot increments), twenty four headings (0 to 345 degrees in 15 degree increments), and four significant wave height/modal period combinations; 1 foot and 7 seconds (Sea State 2), 3 feet and 7 seconds (Sea State 3), 6.2 feet and 9 seconds (Sea State 4), and 10.7 feet and 9 seconds (Sea State 5). For the MCM, each time history is 10 minutes in length at 4 samples per seconds and short crested seas are assumed. Each set of time histories are grouped in data files according to sea state.

Access to the time history data, by the maneuvering simulator, is performed using a data table look-up with linear interpolation between heading and speed as needed. However, it should be understood that linear ship motions theory assumes constant mean heading and constant mean ship speed. Both the transfer functions and generated time histories are sensitive to speed and heading variations. Hence, an interpolated time history for a given mode of motion may not necessarily be the same as a time history derived from a transfer function at that identical speed and heading.

Interpolation between speed and heading is performed using linear blending functions. These functions are dependent only on speed or heading and can therefore be used for all modes of motion without recomputing. For example, the ship speed and relative heading to the waves are given at discrete values $V_1,\ldots,V_N$ and $\lambda_1,\ldots,\lambda_N$ respectively. The blending functions for ship speed

(Φ) and heading (Ψ) when interpolating to speed V and heading λ between the i^{th} and the $i+1^{th}$ speed and j^{th} and $j+1^{th}$ heading are then,

$$\begin{aligned}\Phi(i+1) &= 1-\frac{V-V_{i+1}}{V_{i+1}-V_i}\\ \Phi(i) &= 1-\Phi(i+1)\\ \Psi(j+1) &= 1-\frac{\lambda-\lambda_{j+1}}{\lambda_{j+1}-\lambda_j}\\ \Psi(j) &= 1-\Psi(j+1)\end{aligned} \tag{41}$$

Interpolating at time t takes place between time history values at speeds i and i+1 and heading j and j+1,

$$\begin{aligned}\eta(t, V, \lambda) = {} & [\eta(t, V_{i+1}, \lambda_{j+1}) \bullet \Psi(j+1) + \eta(t, V_{i+1}, \lambda_j) \bullet \Psi(j)] \bullet \Phi(i+1) + \\ & [\eta(t, V_i, \lambda_{j+1}) \bullet \Psi(j+1) + \eta(t, V_i, \lambda_j) \bullet \Psi(j)] \bullet \Phi(i)\end{aligned} \tag{42}$$

Summary

A description of the mathematical model used in developing a simulator for ship maneuvering, stationkeeping, and seakeeping has been presented. The model is based on the concept of a modular ship maneuvering model. This treats the ship hull, rudders, propellers, and bow thruster as individual components, modeled separately using methods appropriate for each component with interaction effects between components accounted for.

Future additions and modification to the model are anticipated. The list includes the addition of roll in the equations of motion, the use vertical axis propellers, towing forces, shallow water effects, ship-ship interaction effects, ship-shore interaction effects, time varying second order wave drift forces, and multiple engine models. Another important modification would be a more rational approach to the superposition of seakeeping motions on the calm water maneuvering motions. The computer program which implements the simulator model might be given automatic control systems for an auto pilot as well as dynamic positionkeeping. New front ends to the simulator will also be developed allowing for a fully graphical, man-in-the-loop simulation.

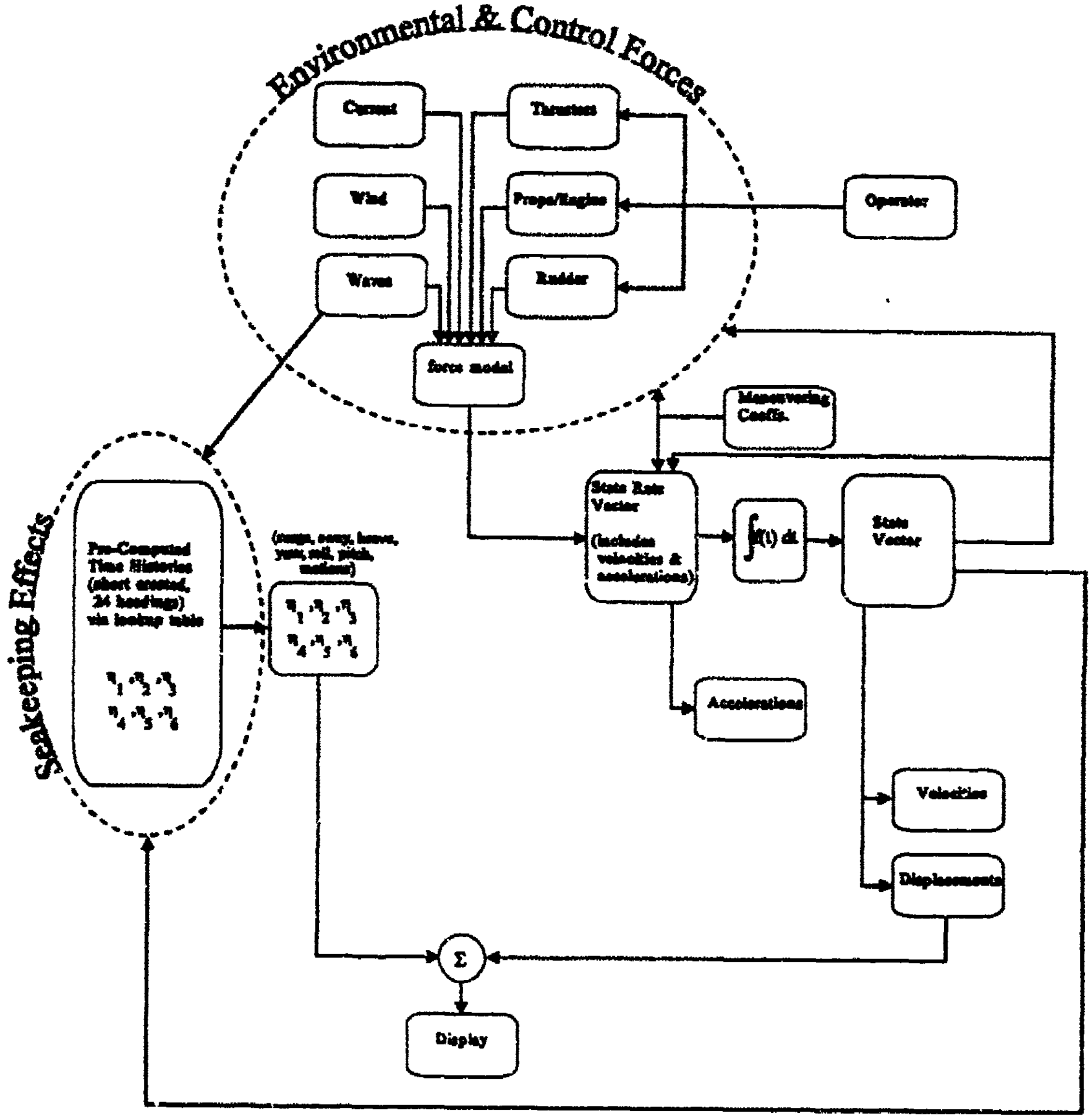

Figure 1 - Conceptual Model

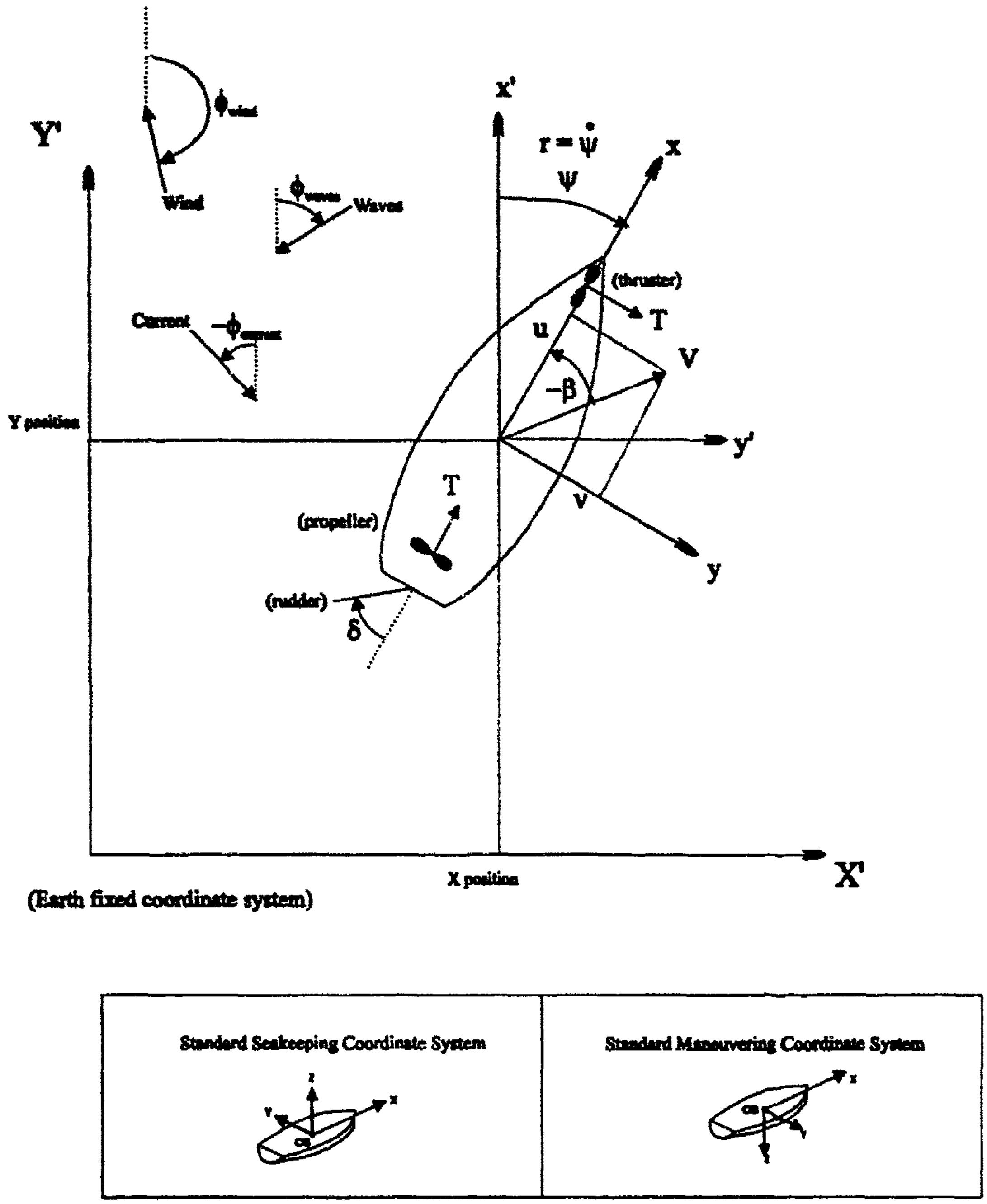

Figure 2 - Coordinate Systems and Sign Conventions

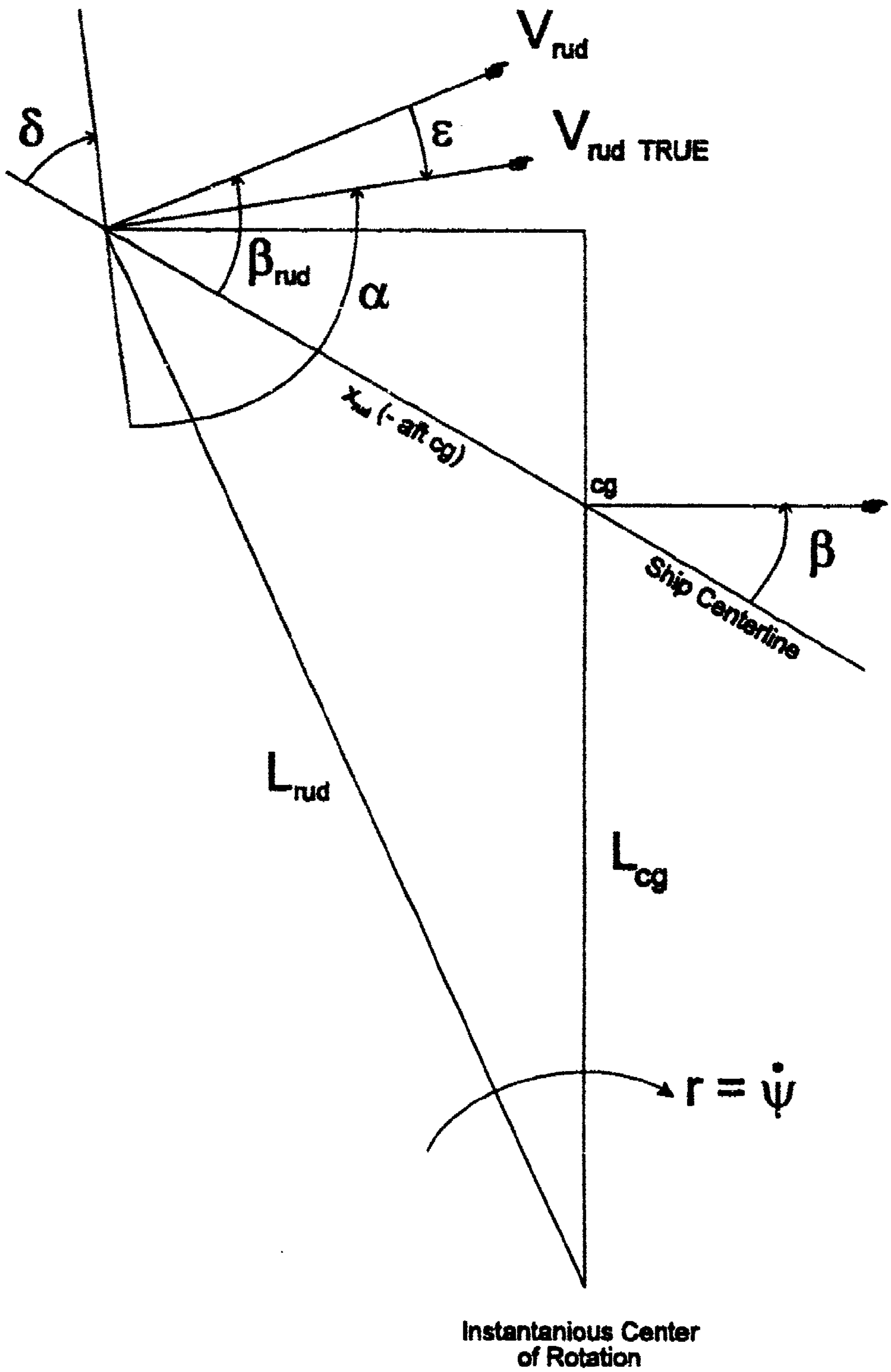

Figure 3 - Rudder Inflow Angle Geometry

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